DISCRETE MATHEMATICS

Section-A

1.Find out which of the following sentences are statements?
(i) May God bless you with success
(ii) Rose is flower
(iii) The colour of the milk is white
(iv) 1 is a prime number.

2. If a compound statement is made up of three simple statements, then the number of rows in the truth table is----------.

3. If *p* is *T* and *q* is *F*, then which of the following have the truth value *T*?
   (i) *p* v *q* (ii) ~ *p* v *q* (iii) *p* v ~ *q* (iv) *p* A ~ *q*

4. The number of rows in the truth table of ~ [p A (~ q)] is

5.The conditional statement $p \rightarrow q$ is equivalent to-------

6. Which of the following is a tautology?
   (1) *p* v *q* (2) *p* A *q* (3) *p* v ~ *p* (4) *p* A ~ *p*

7.Which of the following is a contradiction?
   (1) *p* v *q* (2) *p* A *q* (3) *p* v ~ *p* (4) *p* A ~ *p*

8. *p* <-» *q* is equivalent to
   (1) *p* — *q* (2) *q* — *p* (3) (p — q) v (q — p) (4) (*p* q) A (q — p)

9. Which of the following is not a binary operation on *R*
   (1) $a * b = ab$ (2) $a * b = a - b$
   (3) a * *b* = ▼ab (4) a * *b* = ^/a^2 + b^2

10. A monoid becomes a group if it also satisfies the
   (1) closure axiom (2) associative axiom
   (3) identity axiom (4) inverse axiom

11. Which of the following is not a group?
   (1) (Z_n , $+_n$) (2) (Z, +) (3) (Z, .) (4) (R, +)

12. In the set of integers with operation * defined by $a * b = a + b - ab$, the value of 3 * (4 * 5) is

13. The order of [7] in $(Z_9\ , +_9)$ is

14. In the multiplicative group of cube root of unity, the order of w^2 is

15. The value of [3] $_{+11}$ ([5] $_{+11}$ [6]) is

16. In the set of real numbers R, an operation * is defined by

   a * *b* = ▼a^2 + b^2 . Then the value of (3 * 4) * 5 is

17. The order of - *i* in the multiplicative group of 4th roots of unity is

18. Which of the following is correct?
   (1) An element of a group can have more than one inverse.
   (2) If every element of a group is its own inverse, then the group is abelian.
   (3) The set of all 2 x 2 real matrices forms a group under matrix multiplication.
   (4) (a * b)$^{-1}$ = a"^{1} * b"^{1} for all a, *b* e *G*

19. In the multiplicative group of .nth roots of unity, the inverse of co^k is (k < *n)*

20. In the set of integers under the operation * defined by $a * b = a + b - 1$, the identity element is--------

SECTION- B

1. State and Prove the Cancellation laws .
2. State and Prove the Reversal law.
3. Find the order of each element of the group $(Z_6, +_6)$
4. Construct the truth tables for the following statement $(\sim p) \wedge (\sim q)$
5. Construct the truth tables for the following statement $\sim (p \vee (\sim q))$
6. Construct the truth tables for the following statement $(p \vee q) \wedge (\sim q)$
7. Construct the truth tables for the following statement $\sim [(\sim p) \wedge (\sim q)]$
8. Construct the truth tables for the following statement $(p \vee q) \wedge r$
9. Construct the truth tables for the following statement $(p \wedge q) \vee \sim r$
10. Construct the truth tables for the following statement $(p \vee q) \vee r$
11. Construct the truth tables for the following statement $(p \wedge q) \vee r$
12. Construct the truth tables for the following statement $p \vee \sim q$
13. Construct the truth tables for the following statement$(\sim p) \vee (\sim q)$
14. Construct the truth tables for the following statement $\sim (p \vee q)$
15. Construct the truth tables for the following statement $(p \vee q) \vee (\sim p)$
16. Construct the truth tables for the following statement $(p \wedge q) \wedge (\sim q)$
17. Construct the truth tables for the following statement $(\sim (p \vee (\sim q))$
18. Construct the truth tables for the following statement $(p \wedge q) \vee [\sim (p \wedge q)]$
19. Construct the truth tables for the following statement $(p \wedge q) \vee (\sim q)$
20. Show that $\sim(p \vee q) = ((\sim p) \wedge (\sim q))$
21. Show that $p \wedge \sim p$ is a contradiction.
22. Show that $p \vee \sim p$ is a tautology.
23. Show that $((\sim p) \vee (\sim q)) \vee p$ is a tautology
24. Show that $((\sim q) \wedge p) \wedge q$ is a contradiction.
25. Use the truth table to determine whether the statement $((\sim p) \vee q) \vee (p \wedge (\sim q))$ is a tautology
26. Use the truth table to determine whether the statement $((\sim p) \wedge q)) \wedge p$ is a tautology
27. Use the truth table to determine whether the statement$(p \vee q) \vee (\sim (p \vee q))$ is a tautology
28. Use the truth table to determine whether the statement$(p \wedge (\sim q)) \vee ((- p) \vee q)$) is a tautology
29. Use the truth table to determine whether the statement $q \vee (p \vee (\sim q))$ is a tautology
30. Use the truth table to determine whether the statement $(p \wedge (\sim p)) \wedge ((- q) \wedge p)$

is a tautology.

31. Show that $p \rightarrow q = (\sim p) \vee q$
32. Show that $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$
33. Show that $p \leftrightarrow q = ((- p) \vee q) \wedge ((- q) \vee p)$
34. Show that $\sim(p \wedge q) = ((\sim p) \vee (\sim q))$
35. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent
36. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.
37. Show that the cube roots of unity forms a finite abelian group under multiplication.
38. Prove that the set of all 4^{th} roots of unity forms an abelian group under multiplication
39. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ form an abelian group, under multiplication of matrices.

SECTION- C

1. Show that the set G of all positive rationals forms a group under the composition *defined by
   a*b = ab/3 for all a,b $\in$ G.
2. Show that the set *G* of all rational numbers except - 1 forms an abelian group with respect to the operation * given by $a * b = a + b + ab$ for all a, $b \in$ G.
3. Show that the set *G* of all matrices of the form , wher *e R* - {0}, is a group under matrix multiplication.
4. Show that the set *G* = {2" / *n* $\in$ Z} i s a n abelian group under multiplication

5. Find the order of each element in the group (Z7 - {[0]}, .7)

6. Show that the set {[1], [3], [4], [5], [9]} forms an abelian group under multiplication modulo 11.

7 Prove that the set of four functions /1, /2, /3, /4 on the set of nonzero complex numbers ***C*** - {0} defined by $f_1(z) = z$, $f_2(z) = -z$, $f_3(z) = z$ and $f_4(z) = -z$ $\forall z$ e **C** - {0} forms an abelian group with respect to the composition of functions.

8. Show that $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \right\}$

Where w3 = 1, w 1 form a group with respect to the matrix multiplication.

9. Show that $(Z_n, +_n)$ forms group.

10. Show that the nth roots of unity form an abelian group of finite order with usual multiplication.